Problem sheet 7

7.1 Show that the statement of problem 6.3 c) is not true in higher dimensions:
For $n \geq 2$, there exists a domain $D \subset \mathbb{C}^n$ such that the function
$$u : D \to [-\infty, \infty), \quad u(z) := -\log(\text{dist}(z, \partial D)),$$
is not subharmonic on $D$, where $\text{dist}(z, \partial D) := \inf_{u \in \partial D} \|z - u\|_2$ and $\|\cdot\|_2$ denotes the Euclidean norm.

7.2 For the domains $\Omega$ and functions $u$ defined in $\Omega$ in the following list, determine whether $u$ is harmonic, pluriharmonic or the real part of a holomorphic function. In the latter case, find a corresponding holomorphic function.

a) $\Omega = \mathbb{C}^2$ and $u(x_1 + iy_1, x_2 + iy_2) = x_1^2 - x_2^2$.

b) $\Omega = \mathbb{D} \setminus \{0\} \times \mathbb{D}$ and $u(x_1 + iy_1, x_2 + iy_2) = \frac{x_1}{x_1^2 + y_1^2} + x_2$.

c) $\Omega = \mathbb{C}^2 \setminus \{(0,0)\}$ and $u(x_1 + iy_1, x_2 + iy_2) = x_1 \cdot y_2^2$.

7.3 a) Let $\Omega \subset \mathbb{C}^n$ be a domain. Show: If $\{u_j\}_{j \in J}$ is a family of plurisubharmonic functions on $\Omega$ and $u : \Omega \to [-\infty, \infty), \ u(z) := \sup_{j \in J} u_j(z)$, is upper semicontinuous, then $u$ is plurisubharmonic.

b) Let $\Omega \subset \mathbb{C}^n$ be a domain and let $f \in \mathcal{O}(\Omega)$ be not identically zero. Show that $u : \Omega \to [-\infty, \infty), \ u(z) := \log |f(z)|$, is plurisubharmonic.

7.4 Show that the function $u : \mathbb{C}^2 \to [-\infty, \infty), \ u(z_1, z_2) = \log(|z_1| + |z_2|)$, is plurisubharmonic.

Solutions: On Mon, Dec 09, 16:00 – 17:30.

A small change in the program:
Mon, Dec 16, and Tue, Dec 17: Lecture.
Wed, Dec 18: Problem session.
Mon, Dec 23: nothing.

The final exam will take place on Wed, January 29, in room SE 40.