

OLIVER ROTH

Research in Groups: Complex Analysis Semigroups and Loewner Evolution

(Summer 2016)

- **Class Hours:** Will be fixed at our first meeting on April 11, 4:15 pm, SE 40.
- **Topic:** The Loewner differential equation was introduced by Charles Loewner in 1923 in order to study coefficient problems for univalent functions in the unit disk. It has played an important role in the solution of the Bieberbach conjecture by Louis de Branges in 1985. Since the 1990s the Loewner equation has become an indispensable tool in mathematical physics and has revolutionized our understanding of parametrizations of planar fractal-type curves. Wendelin Werner (2006) and Stanislav Smirnov (2010) received fields medals for their work on the stochastic version of the Loewner equation related to mathematical physics.

This Research in Groups course gives an introduction to the modern aspects of semigroups of holomorphic functions and the Loewner equation, and presents some recent applications.

- **Lectures:**
 1. Univalent functions
 2. Kernel convergence
 3. Loewner chains
 4. The Loewner differential equation
 5. Applications

The lectures are based on [16,17,20,22].

- **Seminar Talks**
 1. Boundary behaviour of conformal maps [2,10: Theorem 3.1,2]
 2. Slit mappings and the associated Loewner equation [8]
 3. Semigroups and their infinitesimal generators: Berkson-Porta Theorem [21]
 4. Evolution families [4,6]
 5. The chordal Loewner equation [13,15]
 6. Numerical computations for the Schramm-Loewner Evolution [12]
 7. Univalence criteria [8,17]
 8. Higher dimensions [11]
 9. Proof of the Bieberbach conjecture [5,7,9,19,20]

References

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