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Zero relaxation time limits to isothermal hydrodynamic model for semiconductor

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ABSTRACT

In this paper, we remove the bounded total variation condition on the initial data and the restriction of the concentration of a fixed background charge being a constant in the paper "Relaxation of the Isothermal Euler–Poisson System to the Drift-Diffusion Equations," (Quart. Appl. Math., 58 (2000), 511–521), and obtain the zero relaxation time limits to isothermal hydrodynamic model for semiconductor by using the varying viscosity method.
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1. Introduction

In this paper, we study the relaxation limit of the one-dimensional isothermal Euler–Poisson model for semiconductor devices:

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + P(\rho))_x = \rho E - \frac{\rho u}{\tau}, \\ E_x = \rho - n(x), \end{cases} \tag{1.1}$$

in the region $(-\infty, +\infty) \times (0, \infty)$, with bounded initial data

$$(\rho, u)|_{t=0} = (\rho_0(x), u_0(x)), \quad \lim_{|x|\to\infty} (\rho_0(x), u_0(x)) = (0, 0), \quad \rho_0(x) \geq 0 \tag{1.2}$$

and a condition at $-\infty$ for the electric field

$$\lim_{x\to-\infty} E(x,t) = E_-, \quad \text{for a.e.} \quad t \in (0, \infty), \tag{1.3}$$

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where T, E_- are fixed constants, $\rho \geq 0$ denotes the electron density, the pressure–density relation is $P(\rho) = \rho$, u the (average) particle velocity and E the electric field, which is generated by the Coulomb force of the particles. The given function $n(x)$ represents the concentration of a fixed background charge [1,2] and $\tau > 0$ is the momentum relaxation time. From the physical and engineering point of view, the isothermal case $P(\rho) = \rho$ is very important. The global existence of entropy solutions of (1.1) with $BV(\mathbb{R})$ initial data was obtained in [2,3] by using the Glimm method [4,5], and with bounded $L^\infty(\mathbb{R})$ initial data or initial–boundary values was well studied in [6,7] by using the compensated compactness method.

In this paper, we are concerned with the relaxation limit of the problem (1.1)–(1.3) when $\tau \to 0^+$. In the isentropic case $P(\rho) = \frac{1}{\gamma}\rho^\gamma, \gamma > 1$, Marcati and Natalini introduced a "parabolic scaling" $s := \tau t, x := x$, and showed that in the new variables, the solution converges to the solution of the drift-diffusion system [8] (See also [9,10] for the solutions in $L^p, 1 < p < \infty$ space). In the isothermal case $P(\rho) = \rho$, under the bounded total variation condition on the initial data and the restriction $n(x) = N$, where $N \geq 0$ is a constant, Junca and Rascle [11] proved that the $BV(\mathbb{R})$ solution (ρ^τ, E^τ), obtained in [2] converges to the solution of the drift–diffusion equations

$$\begin{cases} \frac{\partial \rho}{\partial s} + \frac{\partial}{\partial x}(\rho E - \frac{\partial \rho}{\partial x}) = 0 \\ \frac{\partial E}{\partial x} = \rho - N \end{cases} \tag{1.4}$$

in the sense of distributions, where $s := \tau t$ and (ρ, E) is the relaxation limit of (ρ^τ, E^τ) as $\tau \to 0^+$. In this paper, under the assumptions of the initial data $u_0(x) \in L^\infty(\mathbb{R}), \rho_0(x) \in L^\infty(\mathbb{R}) \cap L^1(\mathbb{R})$ and $n(x) \in L^1(\mathbb{R})$, we obtain the similar zero-relaxation limit by using the varying viscosity method.

The classical viscosity method is to add the diffusion terms to the right-hand side of system (1.1) and to study the following parabolic system

$$\begin{cases} \rho_t + (\rho u)_x = \varepsilon \rho_{xx}, \\ (\rho u)_t + (\rho u^2 + P(\rho))_x = (\rho u)_{xx} + \rho E - \frac{\rho u}{\tau}, \\ E_x = \rho - n(x). \end{cases} \tag{1.5}$$

If we consider the momentum $m = \rho u$ as an independent variable, we must first obtain the positive, lower estimate of ρ^ε since $\rho u^2 = \frac{m^2}{\rho}$ in the second equation of (1.5) is not well defined at $\rho = 0$. However, if we apply the third equation in (1.5) to resolve $E^\varepsilon(x,t)$, the new problem arises of how to control the integral of $\int_{-\infty}^x \rho^\varepsilon(x,t)dx$.

To overcome the above difficulty, we construct the approximate solutions of (1.1) by adding the classical viscosity coupled with the flux approximation

$$\begin{cases} \rho_t + ((\rho - 2\delta)u)_x = \varepsilon \rho_{xx}, \\ (\rho u)_t + (\rho u^2 - \delta u^2 + \rho - 2\delta \ln \rho)_x = \varepsilon(\rho u)_{xx} + (\rho - 2\delta)E - \frac{1}{\tau}(\rho - 2\delta)u, \\ E_x = (\rho - 2\delta) - n(x) \end{cases} \tag{1.6}$$

with the initial data

$$(\rho^{\varepsilon,\delta}(x,0), u^{\varepsilon,\delta}(x,0)) = (\rho_0(x) + 2\delta, u_0(x)) * G^\varepsilon, \tag{1.7}$$

where $(\rho_0(x), u_0(x))$ are given in (1.2), $\delta > 0$ denotes a regular perturbation constant, G^ε is a mollifier such that $(\rho^{\varepsilon,\delta}(x,0), u^{\varepsilon,\delta}(x,0))$ are smooth and

$$\lim_{|x|\to\infty} (\rho^{\varepsilon,\delta}(x,0), u^{\varepsilon,\delta}(x,0)) = (2\delta, 0), \qquad \lim_{|x|\to\infty} (\rho_x^{\varepsilon,\delta}(x,0), u_x^{\varepsilon,\delta}(x,0)) = (0,0). \tag{1.8}$$

One obvious advantage of the above viscosity-flux approximation is that we may obtain the bound $\rho^{\varepsilon,\delta} \geq 2\delta > 0$ immediately, by applying the maximum principle to the first equation in (1.7), which guarantees that both the term $\rho u^2 = \frac{m^2}{\rho}$, and the function $E^{\varepsilon,\delta}(x,t) = \int_{-\infty}^x \rho^{\varepsilon,\delta}(x,t) - 2\delta - n(x)dx$ are well defined. More precisely, the following lemma was obtained in [10] by using the compensated compactness method [12]

Lemma 1.1. *Let $(\rho_0(x), u_0(x))$ be bounded in $L^\infty(\mathbb{R})$ and $(\rho_0(x), n(x))$ be bounded in $L^1(\mathbb{R})$. Then, for any fixed $\varepsilon > 0, \delta > 0, \tau > 0$, the problem (1.6)–(1.8) has a unique global smooth solution $(\rho^{\varepsilon,\delta}(x,t), u^{\varepsilon,\delta}(x,t), E^{\varepsilon,\delta}(x,t))$ in $\mathbb{R} \times (0,T]$, satisfying*

$$\begin{cases} \lim_{|x|\to\infty}(\rho^{\varepsilon,\delta}(x,t), u^{\varepsilon,\delta}(x,t)) = (2\delta, 0), \quad \lim_{|x|\to\infty}(\rho_x^{\varepsilon,\delta}(x,t), u_x^{\varepsilon,\delta}(x,t)) = (0,0), \\ \ln \rho^{\varepsilon,\delta}(x,t) - u^{\varepsilon,\delta}(x,t) \le M_1 + M_2 t, \ \ln \rho^{\varepsilon,\delta}(x,t) + u^{\varepsilon,\delta}(x,t) \le M_1 + M_2 t, \\ 0 < 2\delta \le \rho^{\varepsilon,\delta}, \ |\rho^{\varepsilon,\delta}(\cdot,t) - 2\delta|_{L^1(\mathbb{R})} \le M_3, \ |E^{\varepsilon,\delta}| \le M_3, \end{cases} \tag{1.9}$$

where the constants $M_i, i = 1,2,3$ depend only on the bounds of the initial data, but are independent of $\varepsilon, \delta, \tau$.

In this paper, we are concerned with the zero-relaxation-time-limit of above viscosity solutions as $\varepsilon, \delta, \tau$ go to zero, without the uniformly time-independent estimates on $(\rho^{\varepsilon,\delta}, u^{\varepsilon,\delta})$.

Theorem 1.1. *Let the conditions in Lemma 1.1 and $u_0^2(x) \in L^1(\mathbb{R})$ be satisfied; let $s = \tau t, v^\tau(x,s) = v^{\varepsilon,\delta}(x, \frac{s}{\tau}) = v^{\varepsilon,\delta}(x,t)$ for any function v. Then, there exists a subsequence (still labelled) $(\{\rho^\tau\}, \{E^\tau\})$ such that $\rho^\tau(x,s) \to \rho(x,s)$ weakly in $L^1_{loc}(\mathbb{R} \times \mathbb{R}^+)$, $E^\tau(x,s) \to E(x,s)$ strongly in $L^p_{loc}(\mathbb{R} \times \mathbb{R}^+), p \ge 1$ when $\varepsilon, \delta, \tau$ go to zero, and the limit (ρ, E) is a solution of the drift–diffusion equations*

$$\frac{\partial \rho}{\partial s} + \frac{\partial}{\partial x}(\rho E - \frac{\partial \rho}{\partial x}) = 0, \quad \frac{\partial E}{\partial x} = \rho - n(x) \tag{1.10}$$

in the sense of distributions.

Remark 1.1. It is worthwhile to point out that the results in Theorem 1.1 can be easily extended to the following Euler–Poisson equations of two-carrier types in one dimension

$$\begin{cases} \rho_{it} + (\rho_i u_i)_x = 0, \\ (\rho_i u_i)_t + (\rho_i (u_i)^2 + \rho_i)_x = \rho_i E - \frac{\rho_i u_i}{\tau_i}, \quad i = 1,2, \\ E_x = \rho_1 + \rho_2 - n(x), \end{cases} \tag{1.11}$$

in the region $(-\infty, +\infty) \times [0,T]$, with suitable bounded initial data and the condition (1.3) at $-\infty$ for the electric potential E, where (ρ_1, u_1) and (ρ_2, u_2) are the (density, velocity) pairs for electrons ($i = 1$) and holes ($i = 2$) respectively, and the given function $n(x)$ represents the impurity doping profile.

In the above case, the drift–diffusion equations (1.10) are replaced by

$$\frac{\partial \rho_i}{\partial s} + \frac{\partial}{\partial x}(\rho_i E - \frac{\partial \rho_i}{\partial x}) = 0, \ \frac{\partial E}{\partial x} = \rho_1 + \rho_2 - n(x), \quad i = 1,2. \tag{1.12}$$

2. The proof of Theorem 1.1

Let $s = \tau t$ and $v^{\varepsilon,\delta}(x,t) = v^{\varepsilon,\delta}(x, \frac{s}{\tau}) = v^\tau(x,s)$. Then

$$\frac{\partial v^{\varepsilon,\delta}}{\partial t} = \frac{\partial v^\tau}{\partial s}\frac{\partial s}{\partial t} = \tau \frac{\partial v^\tau}{\partial s}, \quad \frac{\partial v^{\varepsilon,\delta}}{\partial x} = \frac{\partial v^\tau}{\partial x} \tag{2.1}$$

and

$$\begin{cases} \tau \frac{\partial \rho^\tau}{\partial s} + \frac{\partial}{\partial x}((\rho^\tau - 2\delta)u^\tau) = \varepsilon \frac{\partial^2 \rho^\tau}{\partial x^2}, \\ \tau \frac{\partial (\rho^\tau u^\tau)}{\partial s} + \frac{\partial}{\partial x}((\rho^\tau - \delta)(u^\tau)^2 + \rho^\tau - 2\delta \ln \rho^\tau) = (\rho^\tau - 2\delta)E^\tau - \frac{1}{\tau}(\rho^\tau - 2\delta)u^\tau + \varepsilon \frac{\partial^2 (\rho^\tau u^\tau)}{\partial x^2}, \\ \frac{\partial E^\tau}{\partial x} = \rho^\tau - b(x) \end{cases} \tag{2.2}$$

due to (1.6).

We shall prove Theorem 1.1 by the following several lemmas.

Lemma 2.1. *We have the estimates*

$$\int_0^L \int_{\mathbb{R}} \frac{1}{\tau^2}(\rho^\tau - 2\delta)(u^\tau)^2 dxds \leq M(L) \quad \textit{and} \quad \int_0^L \int_{\mathbb{R}} \frac{\varepsilon}{\tau\rho^\tau}(\rho_x^\tau)^2 \leq M(L). \tag{2.3}$$

Proof of Lemma 2.1. Multiplying the first equation in (2.2) by $\frac{\partial\eta^\star}{\partial\rho}$, and the second by $\frac{\partial\eta^\star}{\partial m}$, then adding the result, we have

$$\begin{aligned}&\tau\eta_s^\star(\rho^\tau, m^\tau) + q_x^\star(\rho^\tau, m^\tau) = \varepsilon\eta_{xx}^\star(\rho^\tau, m^\tau) + (\rho^\tau - 2\delta)u^\tau E^\tau - \tfrac{1}{\tau}(\rho^\tau - 2\delta)(u^\tau)^2 \\ &-\varepsilon(\rho_x^\tau, m_x^\tau)\cdot\nabla^2\eta^\star(\rho^\tau, m^\tau)\cdot(\rho_x^\tau, m_x^\tau)^T,\end{aligned} \tag{2.4}$$

where

$$\begin{cases} \eta^\star(\rho, m) = \frac{m^2}{2\rho} + \rho(\ln\rho - \ln 2\delta) \geq 0, \\ q^\star(\rho, m) = \frac{m^3}{2\rho^2} + \rho u(\ln\rho - \ln 2\delta) + \rho u - \frac{1}{3}\delta u^3 - 2\delta u - 2\delta u(\ln\rho - \ln 2\delta), \end{cases} \tag{2.5}$$

$$(\rho^\tau - 2\delta)u^\tau E^\tau - \tfrac{1}{\tau}(\rho^\tau - 2\delta)(u^\tau)^2 \leq -\tfrac{1}{2\tau}(\rho^\tau - 2\delta)(u^\tau)^2 + \tfrac{1}{2}\tau(\rho^\tau - 2\delta)(E^\tau)^2 \tag{2.6}$$

and

$$-\varepsilon(\rho_x, m_x)\cdot\nabla^2\eta^\star(\rho, m)\cdot(\rho_x, m_x)^T = -\varepsilon\big((\tfrac{m^2}{\rho^3} + \tfrac{1}{\rho})\rho_x^2 - 2\tfrac{m}{\rho^2}\rho_x m_x + \tfrac{1}{\rho}m_x^2\big) \leq -\varepsilon\tfrac{1}{\rho}\rho_x^2. \tag{2.7}$$

Since

$$\begin{aligned}&\eta^\star(\rho, m)|_{s=0} = \tfrac{1}{2}\rho^{\varepsilon,\delta}(x,0)(u^{\varepsilon,\delta}(x,0))^2 + \rho^{\varepsilon,\delta}(x,0)(\ln\rho^{\varepsilon,\delta}(x,0) - \ln 2\delta) \\ &= \tfrac{1}{2}(\rho_0(x) + 2\delta)u_0^2(x) + (\rho_0(x) + 2\delta)(\ln(\rho_0(x) + 2\delta) - \ln 2\delta) \leq M(u_0^2(x) + \rho_0(x)),\end{aligned} \tag{2.8}$$

which is integrable in $L^1(\mathbb{R})$ by the conditions in Theorem 1.1, and

$$\int_{\mathbb{R}} \frac{1}{2}\tau(\rho^\tau - 2\delta)(E^\tau)^2 dx \leq \tau M \tag{2.9}$$

due to the estimates in (1.9), thus, we may obtain the estimates (2.3) in Lemma 2.1 immediately by using (2.5)–(2.8), if we integrate both sides of (2.4) on $\mathbb{R}\times[0, L]$.

Lemma 2.2. *There exists a subsequence (still labelled)* $\{E^\tau\}$ *such that* $E^\tau(x,s) \to E(x,s)$ *strongly in* $L^p_{loc}(\mathbb{R}\times\mathbb{R}^+)$*, for any* $p \geq 1$*, when* $\varepsilon, \delta, \tau$ *go to zero.*

Proof of Lemma 2.2. Since $|E^\tau(x,s)|_{L^\infty} \leq M, |E_x^\tau(x,s)|_{L^1_{loc}(\mathbb{R}\times\mathbb{R}^+)} \leq M$, then $\frac{\partial c}{\partial s} + \frac{\partial E^\tau}{\partial x}$ are compact in $H^{-1}_{loc}(R\times R^+)$ by using the Murat's lemma [13], where c is an arbitrary constant. Furthermore, by using the third and first equations in (2.2), we have

$$\frac{\partial E^\tau}{\partial s} = \int_{-\infty}^x \frac{\partial\rho^\tau(x,s)}{\partial s}dx = -\frac{(\rho^\tau - 2\delta)u^\tau}{\tau} + \frac{\varepsilon}{\tau}\frac{\partial\rho^\tau(x,s)}{\partial x}, \tag{2.10}$$

which is bounded in $L^1_{loc}(\mathbb{R}\times\mathbb{R}^+)$ because

$$\int_0^L \int_{\mathbb{R}} (\rho^\tau - 2\delta)|u^\tau| dxds \leq \Big(\int_0^L \int_{\mathbb{R}} (\rho^\tau - 2\delta)dxds\Big)^{\frac{1}{2}} \cdot \Big(\int_0^L \int_{\mathbb{R}} (\rho^\tau - 2\delta)(u^\tau)^2 dxds\Big)^{\frac{1}{2}} \leq M\tau \tag{2.11}$$

from the first estimate in (2.3); and for any fixed $N > 0$,

$$\begin{aligned}&\textstyle\int_0^L \int_{-N}^N \frac{\varepsilon}{\tau}|\frac{\partial\rho^\tau(x,s)}{\partial x}|dxds \leq \Big(\int_0^L \int_{-N}^N \frac{\varepsilon}{\tau\rho^\tau}(\rho_x^\tau)^2 dxds\Big)^{\frac{1}{2}} \cdot \Big(\int_0^L \int_{-N}^N \frac{\varepsilon}{\tau}\rho^\tau dxds\Big)^{\frac{1}{2}} \\ &\textstyle\leq M(L)\Big(\int_0^L \int_{-N}^N \frac{\varepsilon}{\tau}e^{M_1 + M_2\frac{s}{\tau}}dxds\Big)^{\frac{1}{2}} \leq M(L)(LN)^{\frac{1}{2}},\end{aligned} \tag{2.12}$$

from the second estimate in (2.3), if we choose ε to be much smaller than τ such that $2\varepsilon \leq \tau e^{-M_1-M_2\frac{L}{\tau}}$. Thus, $\frac{\partial E^\tau}{\partial s}+\frac{\partial c}{\partial x}$ are also compact in $H^{-1}_{loc}(R\times R^+)$ for any constant c. If we apply the Div–Curl lemma [14] to the pairs of functions

$$(c, E^\tau), \quad (E^\tau, c), \tag{2.13}$$

we may obtain

$$\overline{E^\tau}\cdot\overline{E^\tau}=\overline{(E^\tau)^2}, \tag{2.14}$$

which deduces the pointwise convergence of E^τ and the proof of Lemma 2.2.

Proof of Theorem 1.1. Eliminating $\frac{1}{\tau}(\rho^\tau-2\delta)u^\tau$ in the first two equations in (2.2), we obtain

$$\begin{aligned}
&\tfrac{\partial \rho^\tau}{\partial s}+\tfrac{\partial}{\partial x}(\rho^\tau E^\tau-\tfrac{\partial\rho^\tau}{\partial x})=\tfrac{\varepsilon}{\tau}\tfrac{\partial^2\rho^\tau}{\partial x^2}+\varepsilon\tfrac{\partial^2(\rho^\tau u^\tau)}{\partial x^2}-2\delta E^\tau\\
&-\tau\tfrac{\partial(\rho^\tau u^\tau)}{\partial s}-\tfrac{\partial}{\partial x}((\rho^\tau-\delta)(u^\tau)^2-2\delta\ln\rho^\tau)\\
&=\tfrac{\varepsilon}{\tau}\tfrac{\partial^2\rho^\tau}{\partial x^2}+\varepsilon\tfrac{\partial^2(\rho^\tau u^\tau)}{\partial x^2}-2\delta E^\tau-2\delta\tau\tfrac{\partial u^\tau}{\partial s}-\delta\tfrac{\partial}{\partial x}(u^\tau)^2+2\delta\tfrac{\partial}{\partial x}(\ln\rho^\tau)\\
&-\tau\tfrac{\partial}{\partial s}((\rho^\tau-2\delta)u^\tau)-\tfrac{\partial}{\partial x}((\rho^\tau-2\delta)(u^\tau)^2).
\end{aligned} \tag{2.15}$$

Using the estimates in (1.9), we have

$$2\delta\leq\rho^\tau\leq e^{M_1+M_2 t}\leq e^{M_1+M_2\frac{L}{\tau}},\ |u^\tau|\leq M_1+M_2\frac{L}{\tau}+|\ln(2\delta)| \tag{2.16}$$

for $s\in(0,L)$. Thus for fixed L,

$$\frac{\varepsilon}{\tau}\frac{\partial^2\rho^\tau}{\partial x^2}+\varepsilon\frac{\partial^2(\rho^\tau u^\tau)}{\partial x^2}-2\delta E^\tau-2\delta\tau\frac{\partial u^\tau}{\partial s}-\delta\frac{\partial}{\partial x}(u^\tau)^2+2\delta\frac{\partial}{\partial x}(\ln\rho^\tau)\to 0, \tag{2.17}$$

in the sense of distributions, if we choose ε,δ to go zero much faster than τ; and

$$-\tau\frac{\partial}{\partial s}((\rho^\tau-2\delta)u^\tau)-\frac{\partial}{\partial x}((\rho^\tau-2\delta)(u^\tau)^2)\to 0, \tag{2.18}$$

in the sense of distributions, due to (2.11) and the first estimate in (2.3).

Suppose $\rho^\tau\to\rho$ weakly in $L^1_{loc}(\mathbb{R}\times\mathbb{R}^+)$, $E^\tau(x,s)\to E(x,s)$ strongly in $L^p_{loc}(\mathbb{R}\times\mathbb{R}^+), p\geq 1$ when ε,δ,τ go to zero. Then the limit (ρ,E) satisfies the drift–diffusion equations (1.10), in the sense of distributions, if we let ε,δ,τ go to zero in (2.15), and the third equation in (2.2). Theorem 1.1 **is proved.**

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