

# NEWSLETTER

## of the Work Group Mathematical Fluid Mechanics

### Newsletter no. 6 (2021)

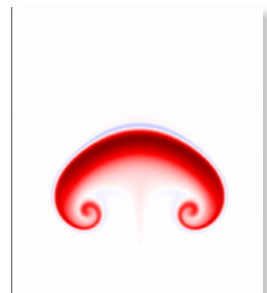
#### Winnie Hartwig submitted her Master thesis

Last week Winnie Hartwig submitted her Master thesis. She studied the numerical approach by Jean-Luc Guermond, who insists that the numerical scheme should maintain all invariant domains of the multi-dimensional system of conservation law at hand. Eva Horlebein is picking up this topic at the start of her PhD thesis.

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#### Overview paper submitted with Fritz Röpke

We have submitted the paper: Edelmann, Horst, Berberich, Andrassy, Higl, Klingenberg, Röpke: "Well-balanced treatment of gravity in astrophysical fluid dynamics simulations at low Mach numbers", [view the PDF of the paper](#).



The joint work with Fritz Röpke has been quite influential for me and the work group. Because of him, we began to study stationary preserving schemes for the Euler equations with gravity, when the mathematical community was still



Fritz Röpke

concentrating on preserving stationary solutions for the shallow

water equations. Our first publications were in 2014. The collaboration with Praveen grew out of this. The list of PhD students who worked on this topic is: Markus Zenk, Wasilij Barsukow, Jonas Berberich and Andrea Thomann. That is why this astrophysics paper submitted last week, feels like a nice milestone: it summarizes some of this work.

*A warm bubble rising in cold air under the influence of gravity. The bubble is a small perturbation of a stationary solution, which the scheme maintains perfectly.*

## Theresa Full begins with her master thesis

Theresa Full will begin her master thesis with us. She plans to work on the Vlasov-Poisson equation

*This is Theresa Full a few years back, still in school in Schweinfurt, where she succeeded in the Math Olympiad.*



## Upcoming scientific conferences

I try to keep this list up-to-date. I encourage you to click the links and check, where you may want to participate.

### 2021:

- March 1 - 5: Oberwolfach: [Hyperbolic Balance Laws: modeling, analysis, and numerics](#), organized among others by Remi Abgrall and Maria Lukacova, to be held hybrid
- March 1 - 5, [SIAM Conference on Computational Science and Engineering](#), online
- March 22- 26: [Kinetic Equations: From Modeling, Computation to Analysis](#), in Marseille, France, organized by Shi Jin, to be held hybrid
- May 24 - 28: [The Legacy of Carlo Cercignani: from Kinetic Theory to Turbulence Modeling](#) Milan, Italy, organized among others by Tommaso Ruggeri
- June 21 - 24: [SIAM Conference on Mathematical & Computational Issues in the Geosciences \(GS21\)](#), in Milan, to be held online
- July 2: [18th International Conference on Hyperbolic Problems, Theory, Numerics, Applications](#) - Part 1 (formerly HYP 2020), one day online with lectures by Dafermos, Mon Tang and others
- July 5 - 9: [THE BOLTZMANN EQUATION: IN THE TRAIL OF TORSTEN CARLEMAN](#), near Stockholm, Sweden
- July 12 - 16: [International Conference on Spectral and Higher Order Methods](#) ICOSAHOM 2020, Vienna (online only), Wasilij and myself plan to run a mini-symposium there
- fall: Special program on Numerical Methods for Nonlinear Hyperbolic PDEs; at SUSTech, Shenzhen, China: organized by Alex Kurganov (still in the planning stage)
- fall: [Nils-Henrik Risebro's birthday conference](#) in Oslo, Norway (still in the planning stage)

### 2022:

- some time in spring: [HIGH ORDER NONLINEAR NUMERICAL METHODS FOR EVOLUTIONARY PDEs: THEORY AND APPLICATIONS \(HONOM\)](#) in Braga, Portugal, organized by Raphael Loubère und Stephane Clain
- end of May: [Sharing Higher-order Advanced Research Know-how on Finite Volume \(SHARK-FV\)](#) in Portugal, organized by Raphael Loubère und Stephane Clain (still in its planning stage)
- June 20 - 25: HYP2022: [18th International Conference on Hyperbolic Problems, Theory, Numerics, Applications](#) - Part 2 (formerly HYP 2020), in Malaga, Spain, organized by Carlos Pares
- July 18 - 22, 2022: [When Kinetic Theory meets Fluid Mechanics](#), Zürich
- Sept. 5 - 9, 2022: [10th International Conference on Numerical Methods for Multi-Material Fluid Flow \(MULTIMAT 2021\)](#) in Zürich, organized by Remi Abgrall