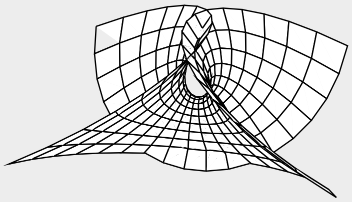

Oberseminar Mathematische Strömungsmechanik

Institut für Mathematik der Julius-Maximilians-Universität Würzburg

Hyperbolic equations - structure preserving methods & other topics

Giovanni Russo

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Conservative Semi-Lagrangian methods for Kinetic Equations

Abstract:

In this talk, we overview a class of high order conservative semi-Lagrangian schemes for BGK model [1-2]. The schemes are constructed by coupling the conservative non-oscillatory reconstruction [1] technique with a conservative treatment of the collision term, obtained by either a discrete Maxwellian [3] or by an L^2-minimization technique [4]. Due to the semi-Lagrangian nature, the time step is not restricted by a CFL-type condition, while the implicit treatment of the relaxation term based on Runge-Kutta or BDF time discretization enables us to avoid the stiffness problem coming from a small Knudsen number. Because of L^2-stability and exact conservation, the resulting scheme is asymptotic preserving for the underlying fluid dynamic limit. Several test cases confirm the accuracy and robustness of the methods, and the AP property of the schemes.

The method has been extended to the treatment of inert gas mixtures, and applied to compare different models in various regimes [5].

In general, such approaches use fixed velocity grids, and one must secure a sufficient number of grid points in phase space to resolve the structure of the distribution function. When dealing with high Mach number problems, where large variation of mean velocity and temperature are present in the domain under consideration, the computational cost and memory allocation requirements become prohibitively large. Local velocity grid methods have been developed to overcome such difficulty in the context of Eulerian based schemes [6-7]. In this talk, we introduce a velocity adaption technique for the semi-Lagrangian scheme applied to the BGK model. The velocity grids will be set locally in time and space. We apply a weighted minimization approach to impose global conservation, generalizing the L^2-minimization technique introduced in [4]. We demonstrate the efficiency of the proposed scheme in several numerical examples.

An additional application of conservative SL schemes concerns the numerical simulation of Vlasov-type equations [8]. Here conservation of the scheme will provide some advantage over standard non conservative schemes for long time computation.

The research is conducted with the following collaborators: S. Boscarino, S. Y. Cho, M. Groppi, S.-B. Yun, JM Qiu, and T. Xiong.

References

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via Zoom video conference (request the Zoom link from klingen@mathematik.uni-wuerzburg.de)

Friday, Oct. 15 at 3 pm CET

Zu diesem Vortrag sind Sie herzlich eingeladen.

gez. Christian Klingenberg