The FEARLESS Cosmic Turbulence Project

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Abstract

We plan to develop, implement, and apply a new numerical scheme for modeling turbulent, multiphase astrophysical flows such as galaxy cluster cores and star forming regions. The method combines the capabilities of adaptive mesh refinement (AMR) and large-eddy simulations (LES) to capture localized features and to represent unresolved turbulence, respectively; it will therefore be referred to as **F**luid m**E**chanics with **A**daptively **R**efined **L**arge-**E**ddy **S**imulation**S** or **FEARLESS**. Recent advances in the field of dynamical subgrid-scale (SGS) models for LES of thermonuclear supernovae enable us to formulate a self-consistent SGS model on adaptive meshes based on local similarity arguments for turbulent transport. Continuing our promising first tests, we intend to implement a full dynamical SGS model into the existing AMR hydrocode ENZO, followed by simulations of turbulent star forming clouds and galaxy cluster turbulence.

1.1 Turbulence in astrophysical phenomena

Many problems of astrophysical hydrodynamics share two important attributes: First, the ubiquitous presence of spatially localized features such as shocks, clumps, or composition discontinuities that need to be numerically resolved or at least adequately modeled; and second, large Reynolds numbers of the baryonic component indicating that fully developed, i.e. space-filling, turbulence is responsible for the mixing and dissipation properties of the gas almost everywhere. Despite great advances in computational fluid dynamics, an accurate handling of both aspects has so far proven to be very difficult as specialized numerical techniques have seemed to be mutually incompatible.

Our project aims at significantly improving this situation. The following three fields are among the most important problems of theoretical astrophysics that our approach may contribute to.

1.1.1 Turbulence in star formation

The efficiency, initial mass function (IMF), and feedback of star formation affects nearly every aspect of theoretical astrophysics. It is well known that turbulence plays a key role in the fragmentation of self-gravitating gas and in its support against gravity [1, 2]. The contribution of turbulence to the statistical properties of the IMF by turbulent fragmentation [3, 4] and to the transport of angular momentum during the collapse [5] are at the center of current theoretical and computational investigations. Furthermore, planned facilities such as ALMA and JWST will enable us to observe the earliest epochs of star formation in the universe.

The irregular shapes of molecular clouds and their complex emission line profiles indicate that the gas motions are supersonic and vigorously turbulent. Supersonic turbulence produces localized structures such as shocks and clumps which are amplified by gravity and cooling instabilities. Resolving these features while simultaneously accounting for the presence of low-amplitude turbulence elsewhere is one of the most challenging problems of computational astrophysics. Many simulations of turbulent star formation have used the Lagrangian method of smoothed particle hydrodynamics (SPH) (see [2] for a review) whose capabilities for modeling turbulence are limited, however, as a result of its numerical diffusivity.

Only recently has adaptive mesh refinement (AMR) been employed for modeling supersonic turbulence in gas clouds [6, 4]. This work has demonstrated the potential of AMR to simulate turbulence in principle, opening a promising new approach that we intend to follow and improve in our project.

Primordial star formation at redshifts $z \approx 30 - 40$ plays a special role both theoretically and observationally because of its relevance for the early chemical and gravitational evolution of galaxies [7]. The pioneering AMR simulations of Abel, Bryan and Norman [8, 5] have shown the importance of angular momentum transport during the collapse and the absence of fragmentation of the central core. It will be interesting to see whether the effect of *turbulence refinement* (sec. 1.2.3) in FEARLESS will have a noticeable impact on these results. Together with supersonic self-gravitating turbulence, the collapse of a primordial gas cloud will provide a well-defined test bed for comparisons of the FEARLESS scheme with previous computations.

1.1.2 Turbulence in galaxy cluster cores

Understanding the evolution of the largest gravitationally bound structures in the universe has been one of the most active and successful fields of research in modern astrophysics. On the theoretical side, much of this progress is a result of the great advances in computational cosmological hydrodynamics, i.e. the numerical modeling of self-gravitational fluids consisting of cold dark matter and baryonic gas, coupled to a variety of relevant physical processes such as radiative cooling, supernova feedback, and chemical enrichment. The core of these models usually consists of an N-body code for the gravitational sector coupled to an Eulerian hydro solver. Great progress has been made regarding the resolution of localized structures using adaptive mesh refinement (AMR) for grid-based solvers or Lagrangian methods like smoothed particle hydrodynamics (SPH) [9, 10, 11]. Turbulence on unresolved scales, on the other hand, has in most cases been neglected in cosmological hydrodynamical simulations.

Examples for the many reasons to improve numerical models of galaxy clusters are the search for a solution of the cooling flow problem, the explanation of the rich substructure of cool cluster cores seen with X-ray instruments such as XMM and CHANDRA, and the fascinating prospect for directly detecting cluster turbulence with the ASTRO-E2 XRS high-resolution spectrometer. We will address each of these problems in the following paragraphs.

X-ray observations of cluster cores have detected less cool gas than predicted by cooling-flow models [12], suggesting that the mass deposition rate of cool gas is suppressed by an unknown mechanism. Various solutions for this problem have been proposed, including feedback from star formation and supernovae [13], turbulence induced by large-scale gas motions [14], and buoyant plasma bubbles from AGN activity [15]. We expect hydrodynamical simulations of the latter two phenomena to be greatly improved by using FEARLESS (due to the *turbulence refinement effect*, sec. 1.2.3) and the level set method for non-diffusive interfaces, respectively.

Observations with X-ray satellites have also shown evidence of spatially localized substructures termed “cool fronts”, “cool bullets”, and “cool filaments” in galaxy cluster cores. Their most likely origin is a combination of radiative cooling and star formation feedback resulting in a picture of hierarchical mergers of cool subclusters [16]. Simulations of such structures involving multiple fluid phases separated by a numerically unresolvable interface are usually limited by the inherent diffusivity of the hydrodynamical scheme. Again, significant improvements can be expected by using interface tracking methods such as the level set scheme proposed in our project.

Finally, the potential detection of turbulent gas motions in their X-ray spectra with the upcoming ASTRO-E2 mission further strengthens the need for highly resolved, self-consistent simulations of cluster turbulence [17, 18]. Comparisons of observations with detailed simulations will help to differentiate between the various proposed mechanisms to ameliorate the cooling flow problem summarized above, all of which are expected to give rise to turbulence.

1.1.3 Modeling of type Ia supernova explosions

Owing, among other things, to their potential as cosmological distance indicators, type Ia supernovae have received widespread attention. Numerous observational campaigns are underway (e.g., the European Supernova Collaboration [19], the ESSENCE project [20], and the Nearby Supernova Factory [21]), allowing us to compare theoretical models with observed spectra and light curves in unprecedented detail.

The present consensus model for supernovae of type Ia is the thermonuclear explosion of a Chandrasekhar mass white dwarf [22]. The burning process proceeds as subsonic deflagration which is driven by turbulence [23, 24]. The production of turbulence is a consequence of Rayleigh-Taylor instabilities due to the lower mass density of nuclear ash as compared to the density of unprocessed material [25].

By means of massively parallel computation on present day supercomputers, the three-dimensional numerical simulation of type Ia supernova explosions has become feasible [26]. However, only the largest dynamical scales can be numerically resolved because the Reynolds numbers are huge and the corresponding number of degree of freedoms is beyond the capability of even the most powerful computers today. Thus, one of the major challenges is to account for the interaction between turbulent eddies and the flame propagation on unresolved length scales. This is achieved by means of a subgrid-scale model [23].

The assumption that type Ia supernovae are the result of a pure deflagration has been challenged because the corresponding numerical models fail to predict certain observational features correctly. Perhaps the most severe problem is the significant amount of unburnt carbon and oxygen at low radial velocities which occur in present deflagration models. Apart from that, the total amount of nickel and the energy output are still short of typical observational values [27, 28]. A delayed detonation scenario has been proposed which apparently resolves these difficulties [29, 30]. The transition from a subsonic deflagration phase to a supersonic detonation is hypothetical, and no plausible mechanism has been found in the particular case of thermonuclear combustion in white dwarfs [31]. On the other hand, new developments in the area of the deflagration model suggest that highly turbulent deflagrations may suffice to match observational features of type Ia supernovae. In particular, advances have been made with more realistic initial conditions, enhanced resolution with non-static grids, and improved subgrid scale models [28, 32] (see also 1.2.2). In any case, more powerful numerical techniques are required in order to settle the issue of the actual explosion mechanism.

1.2 Numerical Modeling Techniques

1.2.1 Adaptive mesh refinement

Considerable progress has been made in modeling as much flow structure as possible by direct numerical computation. The most powerful method for the treatment of non-steady flows exhibiting significant anisotropy and high degrees of intermittency is adaptive mesh refinement (AMR). AMR is based on Eulerian continuum mechanics with a hierarchy of grid patches to approximate the flow on various levels of resolution [33]. In localized regions developing structure on comparatively small length scales, a higher level of refinement is applied while smoother portions of the flow are treated with coarser grids. Astrophysical problems for which AMR is particularly well suited include strong shocks [34] and gravitational collapse [35] among many other applications [10]. All cases exhibit locally steep gradients of the state variables and/or rapid spatiotemporal changes in the velocity field.

Only recently, first steps have been taken to explore the ability of AMR to model fully developed turbulence [6]. The authors have demonstrated that AMR can exploit the fact that dissipation is concentrated in spatial regions of dimension less than three at any instant of time. This is true even for homogeneous turbulence which is commonly considered as "space filling" based on the properties of its ensemble average. Although a detailed comparison of turbulence modeled with AMR and uniform grid simulations remains to be done, it is already clear that AMR is one of the few candidates to significantly extend the range of numerically achievable Reynolds numbers.

However, in the case of astrophysically relevant Reynolds numbers unmanageable levels of refinement would be required in order to resolve the dissipative scales. Moreover, AMR suffers from an intrinsic inconsistency when applied to turbulent flows: As new levels of refinement are added, the velocity on the smallest scales is initially smooth, in contrast with the physical requirement of velocity fluctuations on all scales above the viscous (Kolmogorov) scale. As sketched in fig. 1, this inconsistency will be resolved in FEARLESS by using the subgrid-scale turbulence as an energy buffer (sec. 1.2.3).

1.2.2 Large eddy simulations

In engineering applications as well as other fields of computational fluid dynamics, large eddy simulations (LES) have been in use over several decades [36]. There is no hierarchy of dynamically changing grid patches but one static grid that may be equidistant or unstructured. The conservation laws of hydrodynamics are solved by means of spectral or finite-volume methods. Subgrid scale eddies of size smaller than the grid resolution cannot be resolved by the numerical scheme. The stress exerted by subgrid scale eddies onto resolved eddies that results in the transfer of kinetic energy from larger towards smaller scales is accounted for by a heuristic model, a so-called subgrid-scale (SGS) model [37]. An exact treatment of subgrid scale turbulence is impossible due to the non-linearity in the dynamical equations which entails the fundamental *closure problem* in turbulence theory.

The selection of an appropriate SGS model is a notorious problem because of the considerable variety of models that have been proposed. In astrophysical fluid dynamics, many researchers choose the minimal solution of letting numerical dissipation drain kinetic energy from the resolved flow. Engineers and atmospheric scientists, on the other hand, have recognized that numerical dissipation is an insufficient solution at best [38, 39]. Turbulent burning in thermonuclear supernova explosions is one of the few examples in astrophysics where a

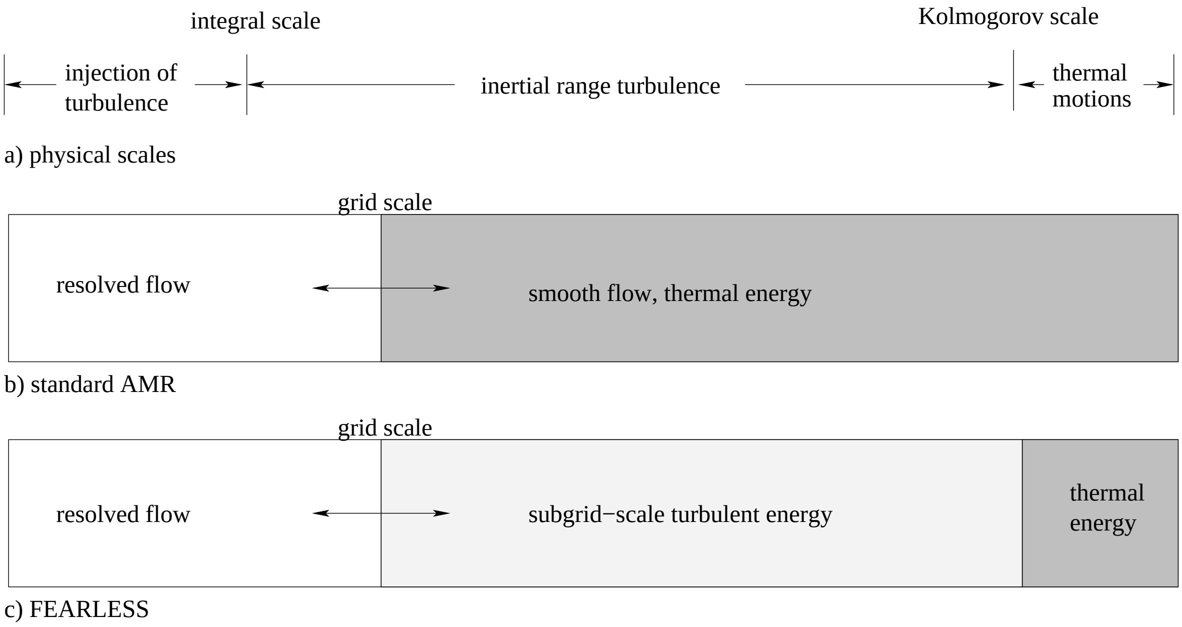

Abbildung 1: Comparison of the physical length scales of fully developed turbulence (a) with the partition between resolved kinetic energy, unresolved kinetic energy, and thermal energy without (standard AMR, (b)), and with (FEARLESS, (c)) an SGS model for unresolved turbulence. The arrows at the grid scale in (b) and (c) represent the variability of the grid scale in AMR and are meant to clarify the *turbulence refinement effect*.

proper SGS model has been applied so far [23, 40].

LES is limited, however, in its representation of highly transient and localized phenomena. This problem can be traced back to the fact that all SGS models are in some way based on the notion of similarity. In other words, an LES must resolve the flow to an extent that turbulence becomes nearly isotropic and scale-invariant towards the smallest resolved length scales. The anisotropy of the flow stemming from large-scale features, such as boundary conditions and mechanical forces, is then mostly confined in the range of "large eddy scales". Under the conditions mentioned above, however, this often becomes infeasible using a static grid approach. In this case, AMR would do a much better job apart from the drawback that it fails to account for small-scale turbulence.

1.2.3 Combining AMR and LES

Few attempts has been made so far to apply AMR in combination with a subgrid-scale (SGS) model. The outcome would be a FEARLESS scheme with locally adapting resolution (for a recent proposal along these lines, see [41]). The central idea is to track turbulent regions in the flow and to explicitly compute any transient or anisotropic features by means of AMR. For each grid patch, an SGS model with the corresponding cutoff scale must then be invoked in order to capture the effect of yet smaller, unresolved velocity fluctuations. The following conceptual and technical questions have kept the development of FEARLESS from progressing very far until now:

1. How can the central similarity arguments of LES that are usually formulated in Fourier

space be transferred to a grid with locally varying resolution?

2. How can filters be implemented self-consistently across AMR patches?
3. How does subgrid-scale turbulence propagate between regions of varying resolution?
4. How can grid refinement and derefinement be made consistent with the presence of turbulent velocity fluctuations that become resolved or unresolved?

Owing to our experience using localized SGS models in supernova Ia research [40, 42, 43], we are now able to answer questions 1 and 2, while 3 and 4 are currently under investigation.

We expect the most noticeable difference between FEARLESS and standard AMR simulations to be the *turbulence refinement effect* related to point 4 above. Assuming the existence of a turbulent cascade on all scales above the (unresolvable) Kolmogorov length, the addition of grid patches must account self-consistently for the kinetic energy of previously unresolved fluctuations that become a part of the resolved flow (cf. fig. 1). We intend to implement this effect by explicitly forcing fluctuations on the newly created small scales and subtracting the corresponding amount of energy from the subgrid-scale energy of the SGS model. The inverse process of derefinement will be handled accordingly by adding the appropriate kinetic energy to the SGS energy. Hence, the SGS model can be used as an energy buffer for grid resolution changes, in addition to its usual application to mimic turbulent mixing and dynamical pressure.

In situations with a large number of refinement levels, as typically encountered in the examples of sec. 1.1, the *turbulence refinement effect* can potentially become the dominant source of turbulent fluctuations on small scales. This is one of our main motivations for revisiting these problems with a FEARLESS approach.

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