



Oberseminar Mathematische Strömungsmechanik

Institut für Mathematik der Julius-Maximilians-Universität Würzburg

Hyperbolic equations - structure preserving methods & other topics

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Innovative Finite Volume methods for solving multi-dimensional hyperbolic systems of conservation laws

Abstract:

In this work, we propose to reuse the notion of simple Riemann solver in Lagrangian coordinates following Gallice "Positive and entropy stable Godunov-type schemes for gas dynamics and MHD equations in Lagrangian or Eulerian coordinates" in Numer. Math., 94, 2003, to develop a new Eulerian Finite Volume scheme in the two-dimensional case on unstructured meshes.

This work is a proof of concept, and we derive the associated first-order accurate cell-centered Eulerian scheme for compressible flows using the Lagrangian to Eulerian correspondence. First the Lagrangian simple Riemann solver is used as a building block to construct its Eulerian counter-part. This solver inherits by construction the properties of the Lagrangian one, mainly: positivity, preservation, entropy dissipation, well-defined CFL condition and wave-speed ordering. From this Riemann solver, a classical two-point first-order Finite Volume Eulerian scheme can be deduced, for which the numerical fluxes of a given cell are computed only with respect to a restricted set of neighbors through a common face.

Next, we introduce another Eulerian numerical scheme, which involves a multi-dimensional Lagrangian nodal solver leading to the so-called multi-point Riemann solver which considers all surrounding cells, including corner cells. The conservation is no more funded on a one-to-one flux cancellation across a face like for most Finite Volume approach. Conversely, conservation is retrieved on a node basis. An associated first-order Eulerian scheme is derived on the basis of this multi-point Riemann solver. We prove that this multi-point scheme still inherits the good properties of the two-point scheme with the extra-property of coupling all neighbor cells in a consistent way.

A large set of numerical results on general unstructured grids are presented on several classical 2D test cases. They show that the two-point scheme suffers from spurious instabilities such as the infamous Carbuncle phenomena, while the multi-dimensional character of the multi-point scheme seems unsusceptible to those.

This is joint work with Pierre-Henri Maire (CEA-Cesta, France).

via Zoom video conference (request the Zoom link from klingen@mathematik.uni-wuerzburg.de)

Friday, Dec. 3 at 3 pm CET

Zu diesem Vortrag sind Sie herzlich eingeladen.

gez. Christian Klingenberg