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Global entropy solutions for systems modelling polymer flooding in enhanced oil recovery

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ABSTRACT

In this paper, we obtain the existence of global entropy solutions for the Cauchy problem of the nonstrictly hyperbolic systems modelling polymer flooding in enhanced oil recovery, under a more flexible condition on the function $\beta(T)$, which models the adsorption of the polymer on rock. This work improves the previous results in the paper Lu (2013), where $\beta(T)$ is limited to meas$\{T : \beta''(T) = 0\} = 0$ or $\beta(T) = bT$ for a nonnegative constant b.

1. Introduction

In this short paper, we are concerned with the existence of entropy solutions of the Cauchy problem for the nonstrictly hyperbolic systems modelling polymer flooding in enhanced oil recovery

$$\begin{cases} S_t + f(S,T)_x = 0, \\ (ST + \beta(T))_t + (Tf(S,T) + \alpha(T))_x = 0, \end{cases} \tag{1.1}$$

with bounded measurable smooth initial data

$$(S(x,0), T(x,0)) = (S_0(x), T_0(x)), \quad 0 \le S_0(x) \le \bar{S}, \lim_{|x|\to\infty} S_0(x) = \bar{S}, \tag{1.2}$$

where $\bar{S}$ is a positive constant.

System (1.1) first appeared in [1], and the existence of a weak solution of the Cauchy problem (1.1)–(1.2) was well studied in [2–16] when $\beta(T) = 0$ and $\alpha(T) = 0$. When $\alpha(T) = 0$ and $\beta(T) \neq 0$, the Riemann problem of (1.1) was resolved in [17].

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The existence of a weak solution of the Cauchy problem (1.1)–(1.2) for general functions $\beta(T)$ and $\alpha(T)$ was proved in [18] when the following conditions (A) and (B), or (A) and (C) on $f(S,T), \alpha(T)$ and $\beta(T)$ are satisfied (A) meas $\{S : f_{SS}(S,T) = 0\} = 0$ for any fixed T; (B) $\beta'(T) \geq 0$, meas $\{T : \beta''(T) = 0\} = 0$ or $\beta(T) = bT, b > 0$ or (C) $\beta(T) = 0$ and $\alpha(T) = 0$.

In this paper, instead of the condition (B) or (C), we obtain the existence results under more flexible conditions on $\alpha(T)$ and $\beta(T)$:

(D) $\alpha(T) \in C^1, \beta(T) \in C^2, \beta'(T) \geq 0, |T\alpha'(T)| \leq M(\delta + \beta'(T))$ and $|\beta''(T)| \leq M, |T\beta''(T)| \leq M(\delta + \beta'(T))$ for a positive constant M and any fixed $\delta > 0$, where M could depend on T in any compact set of $(-\infty, \infty)$.

Remark 1. It is easy to check that the following functions $\alpha(T), \beta(T)$ satisfy the condition (D):

$$\alpha(T) = \begin{cases} 0, & \text{for} \quad T \leq 0, \\ \frac{1}{\alpha_0} T^{\alpha_0}, & \text{for} \quad 0 \leq T \leq 1, \\ g(T), & \text{for} \quad T > 1 \end{cases} \tag{1.3}$$

and

$$\beta(T) = \begin{cases} 0, & \text{for} \quad T \leq 0, \\ \frac{1}{\beta_0} T^{\beta_0}, & \text{for} \quad 0 \leq T \leq 1, \\ T - 1 + \frac{1}{\beta_0}, & \text{for} \quad T > 1, \end{cases} \tag{1.4}$$

where $g(T)$ is an arbitrary C^1 smooth function satisfying $g(1) = \frac{1}{\alpha_0}, g'(1) = 1$ and α_0, β_0 are constants satisfying $\alpha_0 \geq \beta_0 - 1, \beta_0 \geq 2$.

Mainly, we have the following existence results in this paper.

Theorem 1.1. *Suppose $(S_0(x), T_0(x))$ are bounded, $0 \leq S_0(x) \leq \bar{S}$ and $|T_0(x)| \leq M$; $\ln \bar{S} - \ln S_0(x) \in L^1(R)$ and $T_0(x)$ is of bounded total variation; $\alpha(T), \beta(T)$ are suitable smooth functions satisfying (D); $f(S,T)$ satisfies (A) and $f(\bar{S},T) = 0, |\frac{f(S,T)}{S}| \leq M, |\frac{f_T(S,T)}{S}| \leq M$. Then the Cauchy problem (1.1) and (1.2) has a weak entropy solution.*

2. The proof of Theorem 1.1

To prove Theorem 1.1. we first study the smooth solutions for the following parabolic system

$$\begin{cases} S_t + (\frac{(S-\delta)}{S} f(S,T))_x = \varepsilon S_{xx}, \\ (ST + \beta(T))_t + (\frac{(S-\delta)}{S} T f(S,T) + \alpha(T))_x = \varepsilon (ST + \beta(T))_{xx}, \end{cases} \tag{2.1}$$

with initial data

$$(S^{\varepsilon,\delta}(x,0), T^{\varepsilon,\delta}(x,0)) = (\delta + \frac{\bar{S} - \delta}{\bar{S}} S_0(x), T_0(x)), \tag{2.2}$$

where ε, δ are positive, small perturbation constants.

Then

$$\begin{cases} \delta \leq S^{\varepsilon,\delta}(x,0) \leq \bar{S}, \ \lim_{|x|\to\infty} S^{\varepsilon,\delta}(x,0) = \bar{S} \\ 0 \leq \ln \bar{S} - \ln S^{\varepsilon,\delta}(x,0) \leq \ln \bar{S} - \ln S_0(x) \in L^1(R). \end{cases} \tag{2.3}$$

Lemma 2.1. *If the conditions in Theorem 1.1 are satisfied, then for fixed $\varepsilon > 0, \delta > 0$, the global smooth solution $(S^{\varepsilon,\delta}(x,t), T^{\varepsilon,\delta}(x,t))$, of the Cauchy problem (2.1) and (2.2) exists, and satisfies*

$$\delta \leq S^{\varepsilon,\delta}(x,t) \leq \bar{S}, \ |T^{\varepsilon,\delta}(x,t)| \leq M, \ \lim_{|x|\to\infty} S^{\varepsilon,\delta}(x,t) = \bar{S}, \ \textit{for fixed } t, \tag{2.4}$$

$$\int_{-\infty}^{\infty} |T_x^{\varepsilon,\delta}|(x,t)dx \le \int_{-\infty}^{\infty} |T_x^{\varepsilon,\delta}|(x,0)dx \le M, \tag{2.5}$$

and

$$\int_{-\infty}^{\infty} \ln \bar{S} - \ln S^{\varepsilon,\delta}(x,t)dx + \varepsilon \int_0^t \int_{-\infty}^{\infty} \frac{1}{(S^{\varepsilon,\delta})^2}(S_x^{\varepsilon,\delta})^2 dxdt \le |\ln \bar{S} - \ln S_0(x)|_{L^1(R)} + Mt. \tag{2.6}$$

Proof of Lemma 2.1. Since $\delta \le S_0^{\varepsilon,\delta}(x) \le \bar{S}$ and $f(\bar{S},T) = 0$, by applying the maximum principle to the first equation in (2.1), we have $\delta \le S^{\varepsilon,\delta}(x,t) \le \bar{S}$.

Substituting the first equation in (2.1) into the second, we may rewrite the second equation in (2.1) as

$$T_t + \frac{\frac{(S-\delta)}{S}f + \alpha'(T)}{S + \beta'(T)}T_x = \varepsilon T_{xx} + \varepsilon \frac{2S_x + \beta''(T)T_x}{S + \beta'(T)}T_x. \tag{2.7}$$

Then we have the estimates $|T^{\varepsilon,\delta}(x,t)| \le M$ by applying the maximum principle to (2.7).

Thus the existence of the viscosity solution for the Cauchy problem (2.1)–(2.2) can be obtained by the standard theory of semilinear parabolic systems (cf. [18]).

By using (2.7) and a technique from [19] or [18], we may obtain the proof of (2.5). To prove (2.6), multiplying the first equation in (2.1) by $-\frac{1}{S}$, we have

$$(\ln \bar{S} - \ln S)_t - \frac{1}{S}(\frac{(S-\delta)}{S}f(S,T))_x = (\ln \bar{S} - \ln S)_{xx} - \varepsilon \frac{1}{S^2}S_x^2, \tag{2.8}$$

where

$$\begin{aligned} &\tfrac{1}{S}(\tfrac{(S-\delta)}{S}f(S,T))_x = \tfrac{1}{S}(\tfrac{(S-\delta)}{S}f(S,T))_S S_x + \tfrac{1}{S}\tfrac{(S-\delta)}{S}f(S,T)_T T_x \\ &= (\textstyle\int_{\bar{S}}^S \tfrac{1}{\tau}(\tfrac{(\tau-\delta)}{\tau}f(\tau,T))_\tau d\tau)_x - \textstyle\int_{\bar{S}}^S \tfrac{1}{\tau}(\tfrac{(\tau-\delta)}{\tau}f(\tau,T)_T)_\tau d\tau T_x + \tfrac{1}{S}\tfrac{(S-\delta)}{S}f(S,T)_T T_x \end{aligned} \tag{2.9}$$

and

$$|\int_{\bar{S}}^S \frac{1}{\tau}(\frac{(\tau-\delta)}{\tau}f(\tau,T)_T)_\tau d\tau + \frac{1}{S}\frac{(S-\delta)}{S}f(S,T)_T| \le M. \tag{2.10}$$

So, integrating (2.8) in $R \times [0,t]$, we obtain the proof of (2.6) due to (2.5).

Lemma 2.2. *If the condition (D) is satisfied, then there exists a subsequence (still labelled $T^{\varepsilon,\delta}(x,t)$) such that*

$$T^{\varepsilon,\delta}(x,t) \to T(x,t) \tag{2.11}$$

a.e. on any bounded and open set $\Omega \subset R \times R^+$. Furthermore, if the condition (A) is satisfied, then there exists a subsequence (still labelled $S^{\varepsilon,\delta}(x,t)$) such that

$$S^{\varepsilon,\delta}(x,t) \to S(x,t) \tag{2.12}$$

a.e. on any bounded and open set $\Omega \subset R \times R^+$.

Proof of Lemma 2.2. Since $|\frac{T\beta''(T)}{\delta+\beta'(T)}| \le M$ for any fixed $\delta > 0$, we choose n to be a large odd number such that $|\frac{T\beta''(T)}{\delta+\beta'(T)}| \le n-3$, and multiply (2.7) by nT^{n-1} to obtain

$$\begin{aligned} &(T^n)_t + nT^{n-1}\tfrac{\frac{(S-\delta)}{S}f+\alpha'(T)}{S+\beta'(T)}T_x \\ &= \varepsilon(T^n)_{xx} - \varepsilon n(n-1)T^{n-2}(T_x)^2 + \varepsilon nT^{n-1}\tfrac{2S_x+\beta''(T)T_x}{S+\beta'(T)}T_x. \end{aligned} \tag{2.13}$$

Since

$$\begin{cases} |\frac{\frac{(S-\delta)}{S}f+\alpha'(T)}{S+\beta'(T)}T| \le M, \ nT^{n-1}\frac{2S_x}{S+\beta'(T)}T_x| \le nT^{n-2}(T_x)^2 + nT^n(\frac{S_x}{S})^2, \\ |nT^{n-1}\frac{\beta''(T)T_x}{S+\beta'(T)}T_x| \le n(n-3)T^{n-2}|(T_x)^2, \end{cases} \tag{2.14}$$

we have from (2.13) and (2.14) that

$$\varepsilon nT^{n-2}(T_x)^2 \le -(T^n)_t + nT^{n-2}M|T_x| + \varepsilon(T^n)_{xx} + T^n(\tfrac{S_x}{S})^2. \tag{2.15}$$

Let $K \subset R \times R^+$ be an arbitrary compact set and choose $\phi \in C_0^\infty(R \times R^+)$ such that $\phi_K = 1, 0 \le \phi \le 1$. Multiplying (2.15) by ϕ and integrating over $R \times R^+$, we obtain immediately

$$\varepsilon T^{n-2}(T_x^{\varepsilon,\delta})^2 \quad \text{are bounded in } L^1_{loc}(R \times R^+). \tag{2.16}$$

Thus, the terms in (2.13)

$$nT^{n-1}\frac{\frac{(S-\delta)}{S}f + \alpha'(T)}{S + \beta'(T)}T_x + \varepsilon n(n-1)T^{n-2}(T_x)^2 - \varepsilon nT^{n-1}\frac{2S_x + \beta''(T)T_x}{S + \beta'(T)}T_x \tag{2.17}$$

are bounded in $L^1_{loc}(R \times R^+)$ and

$$\varepsilon(T^n)_{xx} \quad \text{are compact in } H^{-1}_{loc}(R \times R^+), \tag{2.18}$$

which deduce that

$$((T^{\varepsilon,\delta})^n)_t \;\text{ or }\; ((T^{\varepsilon,\delta})^n)_t + c_x \quad \text{are compact in } H^{-1}_{loc}(R \times R^+) \tag{2.19}$$

for any constant c, by using the Murat's lemma (cf. [20]).

Finally, since $((T^{\varepsilon,\delta})^n)_x$ or $c_t + ((T^{\varepsilon,\delta})^n)_x$ are bounded in $L^1_{loc}(R \times R^+)$ and so compact in $H^{-1}_{loc}(R \times R^+)$, we may apply the div–curl lemma in the compensated compactness theory [21] to the following special pairs of functions

$$(c, (T^{\varepsilon,\delta})^n), \quad ((T^{\varepsilon,\delta})^n, c) \tag{2.20}$$

to obtain

$$\overline{(T^{\varepsilon,\delta})^n} \cdot \overline{(T^{\varepsilon,\delta})^n} = \overline{(T^{\varepsilon,\delta})^{2n}}, \tag{2.21}$$

which deduces the pointwise convergence of $(T^{\varepsilon,\delta})^n(x,t) \to T^n(x,t)$ a.e. on any bounded and open set $\Omega \subset R \times R^+$, and so the conclusion (2.11) immediately because n is an odd number, where $\overline{f(\theta^{\varepsilon,\delta})}$ denotes the weak-star limit of $f(\theta^{\varepsilon,\delta})$.

Thanks to the pointwise convergence of $T^{\varepsilon,\delta}(x,t)$, we may use the Div–Curl Lemma on the scalar conservation equation (the first equation in (2.1)) with a space–time discontinuous flux [18,22] to obtain the pointwise convergence of $S^{\varepsilon,\delta}(x,t)$ in (2.12). Lemma 2.2 is proved.

Proof of Theorem 1.1. Letting ε, δ go to zero, we may easily prove from (2.1) that the limit $(S(x,t), T(x,t))$ satisfies

$$\begin{cases} \int_0^\infty \int_{-\infty}^\infty S\phi_t + f(S,T)\phi_x dxdt + \int_{-\infty}^\infty S_0(x)\phi(x,0)dx = 0, \\ \int_0^\infty \int_{-\infty}^\infty ST + \beta(T)\phi_t + (Tf(S,T) + \alpha(T))\phi_x dxdt + \int_{-\infty}^\infty S_0(x)T_0(x)\phi(x,0)dx = 0 \end{cases} \tag{2.22}$$

for all test function $\phi \in C_0^1(R \times R^+)$.

Let $C = ST + \beta(T)$. Since $\beta'(T) \ge 0$, then for any fixed $S \in (0, \bar{S}]$, there exists a smooth, inverse function $T = \theta(S,C)$ and system (2.1) can be rewritten as

$$\begin{cases} S_t + (\frac{(S-\delta)}{S}f(S,\theta(S,C)))_x = \varepsilon S_{xx}, \\ C_t + (\frac{(S-\delta)}{S}\theta(S,C)f(S,\theta(S,C)) + \alpha(\theta(S,C)))_x = \varepsilon C_{xx}. \end{cases} \tag{2.23}$$

Let $(\eta(S,C), q(S,C))$ be any convex entropy–entropy flux pair of the system (cf. [23])

$$\begin{cases} S_t + f(S,\theta(S,C))_x = 0, \\ C_t + (\theta(S,C)f(S,\theta(S,C)) + \alpha(\theta(S,C)))_x = 0. \end{cases} \tag{2.24}$$

Multiplying the first and second equations of the system (2.24) by $\frac{\partial\eta(S,C)}{\partial S}$ and $\frac{\partial\eta(S,C)}{\partial C}$, respectively, then adding the result, we have

$$\begin{aligned}&\eta(S,C)_t+q(S,C)_x-\delta\frac{\partial\eta(S,C)}{\partial S}\Big(\frac{f(S,\theta(S,C))}{S}\Big)_x-\delta\frac{\partial\eta(S,C)}{\partial C}\Big(\frac{\theta(S,C)f(S,\theta(S,C))}{S}\Big)_x\\&=\varepsilon\eta(S,C)_{xx}-\varepsilon(\eta_{SS}S_x^2+2\eta_{SC}S_xC_x+\eta_{CC}C_x^2)\leq\varepsilon\eta(S,C)_{xx}.\end{aligned}\tag{2.25}$$

Let $\frac{\partial\eta}{\partial v_i}$ be the partial derivatives of the function η with respect to the first and the second variable. Then we have

$$\begin{aligned}&\delta\frac{\partial\eta(S,C)}{\partial S}\Big(\frac{f(S,\theta(S,C))}{S}\Big)_x+\delta\frac{\partial\eta(S,C)}{\partial C}\Big(\frac{\theta(S,C)f(S,\theta(S,C))}{S}\Big)_x\\&=\delta\frac{\partial\eta(S,ST+\beta(T))}{\partial v_1}\Big(\Big(\frac{f(S,T)}{S}\Big)_SS_x+\Big(\frac{f(S,T)}{S}\Big)_TT_x\Big)\\&+\frac{\partial\eta(S,ST+\beta(T))}{\partial v_2}\Big(\Big(\frac{Tf(S,T)}{S}\Big)_SS_x+\Big(\frac{Tf(S,T)}{S}\Big)_TT_x\Big)\\&=\delta\frac{\partial}{\partial x}\Big(\int_0^S\frac{\partial\eta(S,ST+\beta(T))}{\partial v_1}\Big(\frac{f(S,T)}{S}\Big)_S+\frac{\partial\eta(S,ST+\beta(T))}{\partial v_2}\Big(T\frac{f(S,T)}{S}\Big)_SdS\Big)\\&-\delta\frac{\partial}{\partial T}\Big(\int_0^S\frac{\partial\eta(S,ST+\beta(T))}{\partial v_1}\Big(\frac{f(S,T)}{S}\Big)_S+\frac{\partial\eta(S,ST+\beta(T))}{\partial v_2}\Big(T\frac{f(S,T)}{S}\Big)_SdS\Big)T_x\\&+\delta\Big(\frac{\partial\eta(S,ST+\beta(T))}{\partial v_1}\Big(\frac{f(S,T)}{S}\Big)_T+\frac{\partial\eta(S,ST+\beta(T))}{\partial v_2}\Big(\frac{Tf(S,T)}{S}\Big)_T\Big)T_x.\end{aligned}\tag{2.26}$$

Since T_x is locally bounded in $L^1(R\times R^+)$, letting ε,δ go to zero, we may prove from (2.25) and (2.26) that the following entropy condition, for the limit functions $(S(x,t),T(x,t))$ and any convex entropy–entropy flux pair (η,q),

$$\int_0^\infty\int_{-\infty}^\infty\eta(S,ST+\beta(T))\phi_t+q(S,ST+\beta(T))\phi_x\phi dxdt\geq 0\tag{2.27}$$

holds, where $\phi\in C_0^\infty(R\times R^+-\{t=0\})$ is a non-negative test function. So, we complete the proof of Theorem 1.1.

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