Oberseminar Mathematische Strömungsmechanik

Institut für Mathematik der Julius-Maximilians-Universität Würzburg

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Compact Runge-Kutta methods for solving hyperbolic conservation laws

Abstract:

Flux Reconstruction (FR) is a class of quadrature free, collocation based high order finite element methods for solving hyperbolic conservation laws. The standard FR schemes provide a spatial discretization resulting in a system of ODEs. In order to obtain a fully discrete high order scheme, the usual approach is to use a multistage Runge-Kutta (RK) method to evolve the system of ODEs to the next time level. This requires the spatial discretization to be performed at every Runge-Kutta stage which necessitates interelement communication in the form of numerical flux computations at each RK stage. Interelement communication is a bottleneck of modern, memory-bound hardware and thus popular alternatives to multistage Runge-Kutta methods are single stage methods like Lax-Wendroff and ADER schemes, which are fully discrete schemes that require a single numerical flux computation for each evolution.

In this talk, we discuss the recently proposed compact Runge-Kutta (cRK) discretization of [Q. Chen, Z. Sun, and Y. Xing, SIAM J. Sci. Comput. 46: A1327-A1351, 2024] which reduces the stencil size by performing interelement communication in only the last RK stage. We propose a compact Runge-Kutta Flux Reconstruction (cRKFR) scheme by interpreting cRK as a procedure to approximate time average fluxes. This interpretation leads to a scheme that requires only one numerical flux computation per time step in contrast to the original cRK scheme that requires as many numerical flux computations as there are RK stages.

After introducing the scheme, we will discuss its application to problems with shocks and other discontinuities. The application of a high order method to such problems requires limiters to minimize Gibbs oscillations. We use a blending limiter that adaptively switches to a subcell based scheme in physical regions where the solution is less regular. Additionally, in order to maintain physical bounds of the solution, we propose a flux limiter that gives a provably admissibility preserving cRKFR scheme. We will also briefly discuss the extension of the method to handle stiff source terms and non-conservative products.

Numerical results involving shocks and small scale shear structures will be presented to validate the accuracy and robustness of the scheme.

room 40.03.003 (Emil Fischer Str. 40)

Donnerstag, Mar. 27, 2025 at 12:30 pm, via Zoom

please request the Zoom link from Christian Klingenberg

Zu diesem Vortrag sind Sie herzlich eingeladen.

gez. Christian Klingenberg