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Strict Deformation Quantization and Noncommutative Quantum Field Theories

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There exist several theoretical frameworks attempting to unify quantum physics with gravity, such as string theory, loop quantum gravity, and quantum field theory (QFT) on noncommutative spacetimes. The last approach is possibly the most modest one of the three: Instead of aiming at a full quantization of gravity, it can be understood as the attempt of setting up a formalism in which certain features, that are expected in extreme situations where both quantum and gravitational effects are strong, appear naturally through a description of the spacetime by noncommutative geometry.

These lectures will discuss certain elements of QFT on noncommutative spaces. As appropriate for a summer school for mathematicians, no background in QFT is required. The aim of the lectures is to highlight and explain the additional features and problems that occur when applying tools of noncommutative geometry to a field-theoretic situation, characterized by the two main principles of covariance and locality.

After starting out from heuristical expectations, we will proceed to the model of Doplicher, Fredenhagen and Roberts [2], and discuss spacetime uncertainty relations. Connecting to the topic of deformation quantization, we will then introduce a variant of Rieffel's strict deformation quantization of C^*-algebras, known as the "warped convolution", and study its properties in some detail [1, 3], in particular with regard to the effect of noncommutative structures on covariance and localization. Time permitting, we will also discuss a model of QFT on noncommutative Minkowski space at finite temperature [4], or localized versions of star products and warped convolutions [5].

References

[1] Buchholz, D., Lechner, G., Summers, S. J.: *Warped Convolutions, Rieffel Deformations and the Construction of Quantum Field Theories.* Comm. Math. Phys. **304** (2011), 95–123.

[2] Doplicher, S., Fredenhagen, K., Roberts, J. E.: *The Quantum structure of space-time at the Planck scale and quantum fields.* Comm. Math. Phys. **172** (1995), 187–220.

[3] Grosse, H., Lechner, G.: *Wedge-Local Quantum Fields and Noncommutative Minkowski Space.* JHEP **11** (2007), 012.

[4] Lechner, G., Schlemmer, J.: *Thermal equilibrium states for quantum fields on noncommutative spacetimes.* arXiv:1503.01639 (2015).

[5] Lechner, G., Waldmann, S.: *Strict deformation quantization of locally convex algebras and modules*. arXiv:1109.5950 (2011).