

1 **SUPPLEMENTARY MATERIALS: ACTIVE FLUX METHODS FOR**
2 **HYPERBOLIC CONSERVATION LAWS – FLUX VECTOR**
3 **SPLITTING AND BOUND-PRESERVATION: ONE-DIMENSIONAL**
4 **CASE ***

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6 **SM1. Additional numerical results.**

7 *Example SM1.1* (Shu-Osher shock-entropy wave interaction). This test is used
8 to check a scheme’s ability to resolve a complex solution with both strong and weak
9 shocks and highly oscillatory but smooth waves. The initial data are

$$(\rho, v, p) = \begin{cases} (3.857143, 2.629369, 10.33333), & \text{if } x < -4, \\ (1 + 0.2 \sin(5x), 0, 1), & \text{otherwise,} \end{cases}$$

11 on the domain $[-5, 5]$ with $\gamma = 1.4$. This test is solved until $T = 1.8$.

12 The reference solution is obtained with the fifth-order WENO finite difference
13 scheme on a mesh of 2000 grid points. The solutions computed with the CFL number
14 0.3 based on the JS and different FVS without limiting on a mesh of 400 cells are
15 displayed in Figure SM1. There are very minor differences between the JS and FVS
16 in the enlarged view. We also check the maximal CFL numbers for each kind of point
17 value update such that the simulation is stable, which are around 0.43, 0.42, 0.43, 0.31
18 for the JS, LLF, SW, and VH FVS. If the power law reconstruction is activated for
19 computing the derivatives in the point value update, the corresponding CFL numbers
20 should be reduced to achieve stability, which are around 0.13, 0.15, 0.16, 0.14.

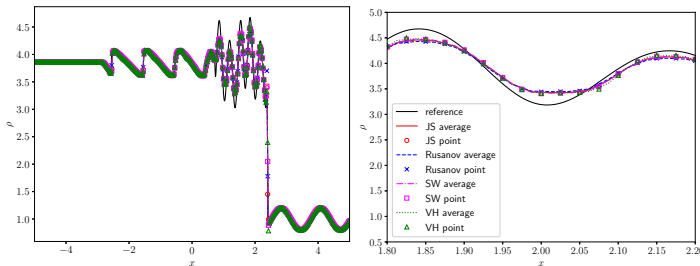


Fig. SM1: Example SM1.1, shock-entropy wave interaction. The numerical solutions are obtained by using the JS and different FVS without limiting on a uniform mesh of 400 cells. The enlarged view in $x \in [1.8, 2.2]$ is shown on the right.

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21 *Example SM1.2* (Double rarefaction problem). Figure SM2 shows the velocity
 22 and pressure computed with 400 cells and the BP limitations for the cell average and
 23 point value updates, while without the power law reconstruction.

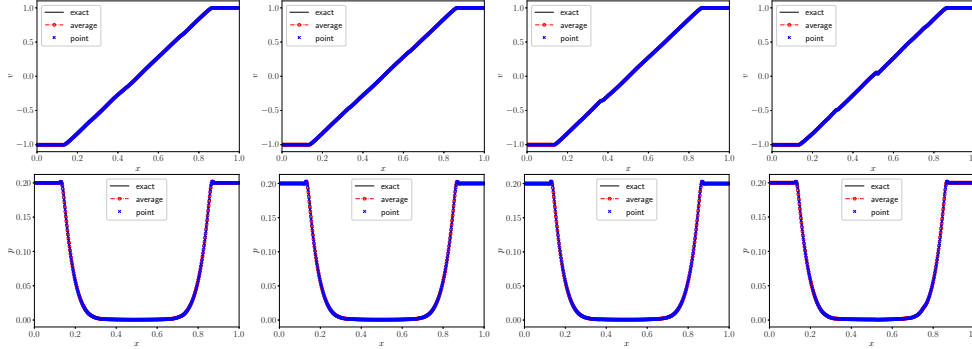


Fig. SM2: Example SM1.2, double rarefaction Riemann problem. The velocity and pressure are computed with BP limitations for the cell average and point value updates on a uniform mesh of 400 cells. The power law reconstruction is not used. From left to right: JS, LLF, SW, and VH FVS.

24 *Example SM1.3* (LeBlanc shock tube). Figure SM3 shows the velocity and pres-
 25 sure computed on a uniform mesh of 400 cells and the BP limitations for the cell average and
 26 point value updates, and Figure SM4 shows the corresponding results with 6000
 27 cells.

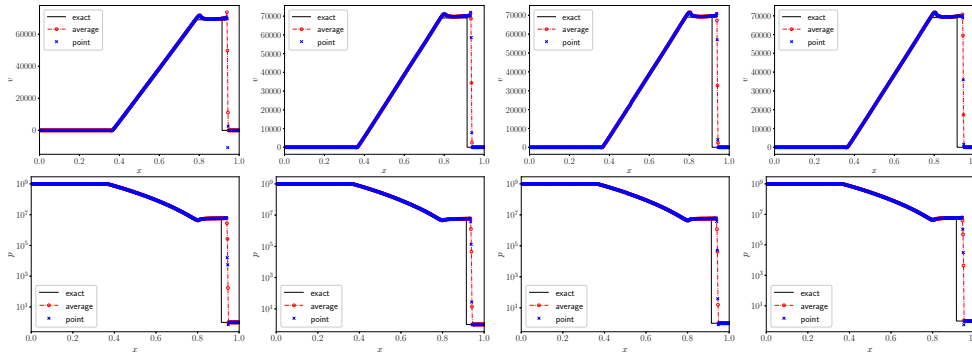


Fig. SM3: Example SM1.3, LeBlanc Riemann problem. The numerical solutions are computed with the BP limitations for the cell average and point value updates on a uniform mesh of 400 cells. The CFL number is 0.4 and the power law reconstruction is not used. From left to right: JS, LLF, SW, and VH FVS.

28 *Example SM1.4* (Blast wave interaction). Figure SM5 shows the density profiles
 29 and corresponding enlarged views in $x \in [0.62, 0.82]$ obtained by using the BP lim-
 30 itings on a uniform mesh of 1600 cells, in which the power law reconstruction is not
 31 activated.

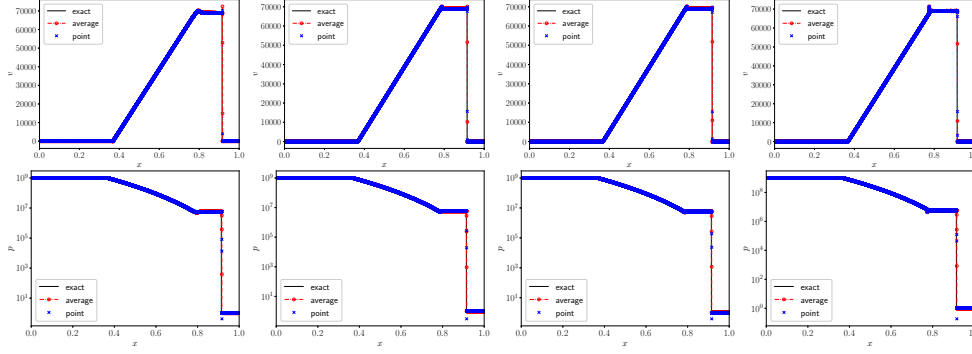


Fig. SM4: Example SM1.3, LeBlanc Riemann problem. The numerical solutions are computed with the BP limitations for the cell average and point value updates on a uniform mesh of 6000 cells. The CFL number is 0.4 and the power law reconstruction is not used. From left to right: JS, LLF, SW, and VH FVS.

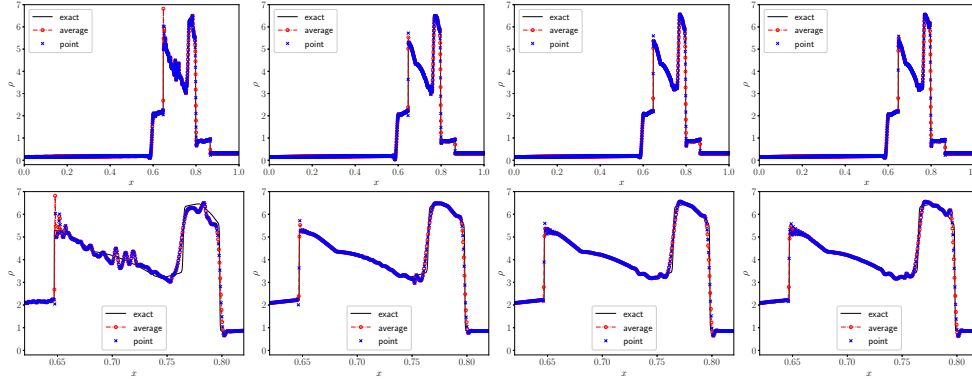


Fig. SM5: Example SM1.4, blast wave interaction. The numerical solutions are computed with the BP limitations for the cell average and point value updates on a uniform mesh of 1600 cells. The power law reconstruction is not used, and from left to right: the CFL number is 0.4, 0.4, 0.4, 0.35 for the JS, LLF, SW, and VH FVS, respectively. The corresponding enlarged views in $x \in [0.62, 0.82]$ are shown in the bottom row.